Radio and Space Plasma Physics

The Leverhulme Trust

Evolution of Poloidal Alfvén Waves in Earth’s Dipole Magnetic Field

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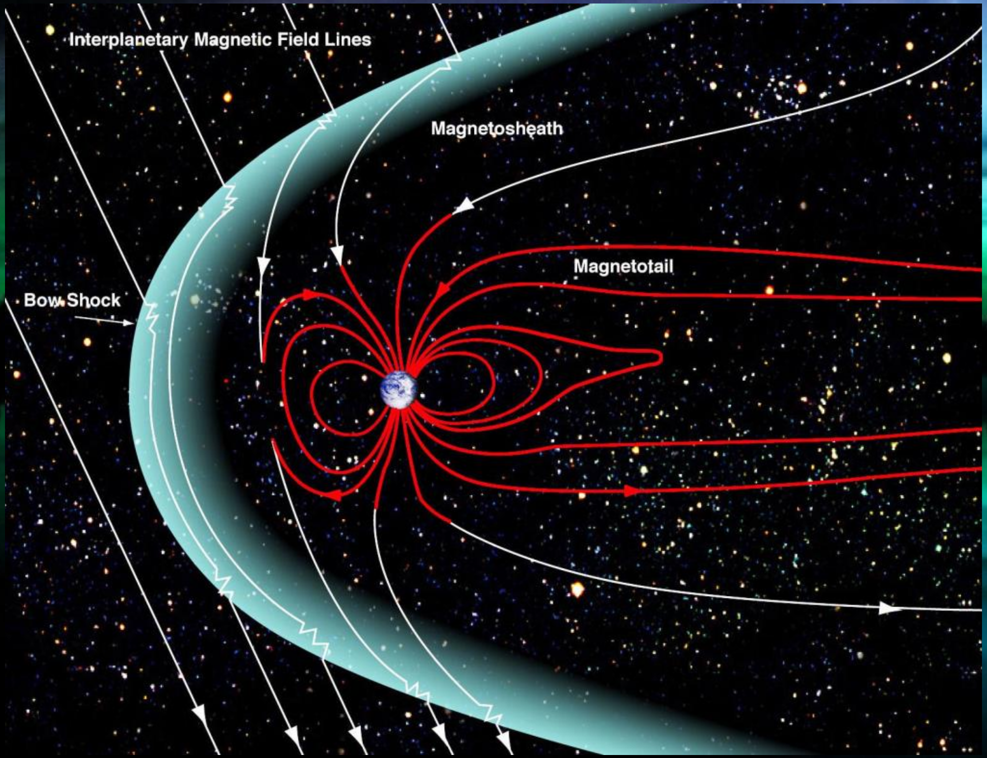


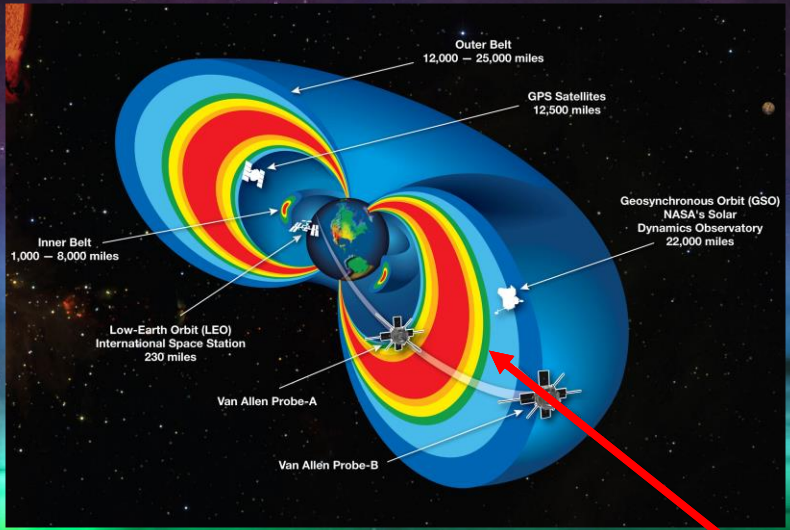
Earth’s Magnetosphere – magnetic field lines in red show the complex connectivity to the interplanetary magnetic field lines in white.

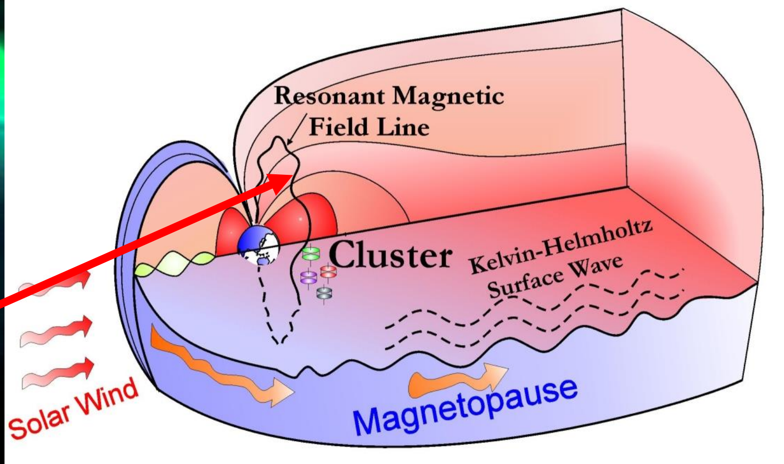
- Earth’s internal magnetic field forms a protective magnetic shield around the Earth, created by the Earth’s magnetic pressure pushing back against the incoming solar wind ram pressure.
- This creates a region inside of which Earth’s magnetic field is dominant, known as the magnetosphere.
- The magnetosphere is filled with natural examples of complex plasma dynamics, such as intricate electrical current systems, highly stressed magnetic field lines and complicated wave-particle interactions.
- Being a system which is constantly perturbed from equilibrium by the dynamic solar wind, *waves* are ubiquitous and transfer energy and momentum throughout the entire magnetosphere.

Ultra-low Frequency (ULF) Waves of Earth’s Magnetosphere

- There are many different wave species of importance in the magnetosphere, but we will discuss here the lowest frequency, largest scale waves called ultra-low frequency (ULF) waves, in the 1mHz-1Hz frequency range.
- These waves represent oscillations of the Earth’s magnetic field and can be measured on the ground or by satellites in space.
- The particular ULF wave of interest for this work is the magnetohydrodynamic (MHD) Alfvén wave (Alfvén, 1942). This propagates along the magnetic field, with a displacement perpendicular to the field, best thought of like a wave on a string.
- In the figure to the right, Alfvén waves would correspond to the curved field line labelled ‘resonant magnetic field line’.

Earth’s radiation belts





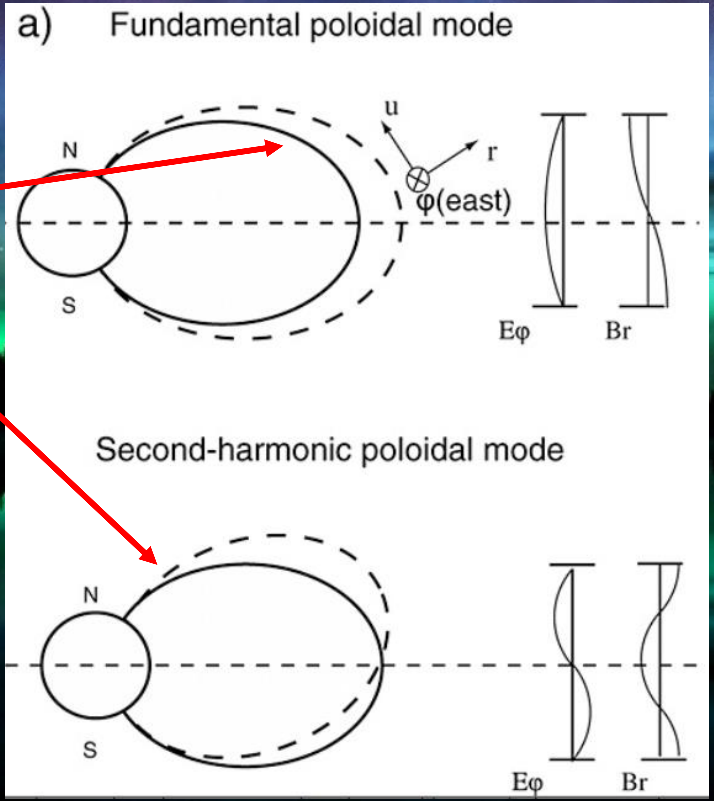
Wave modes of the magnetosphere

- These oscillations are important for many reasons:
- Alfvén waves can be driven resonantly by other wave modes, resulting in a depositing of energy from global to local scales.
- Such resonant Alfvén waves generate local field-aligned currents which can lead to intense auroral displays (e.g Milan+2001).
- They can interact with particles in Earth’s radiation belts shown in the Figure (e.g. Elkington+2003).
- The belts are torus shaped regions where particles are trapped in Earth’s magnetic field.
- Through interactions with ULF waves these particles can be accelerated to incredibly high energies, which is a severe issue for orbiting spacecraft.

Poloidal Alfvén Wave Crash Course

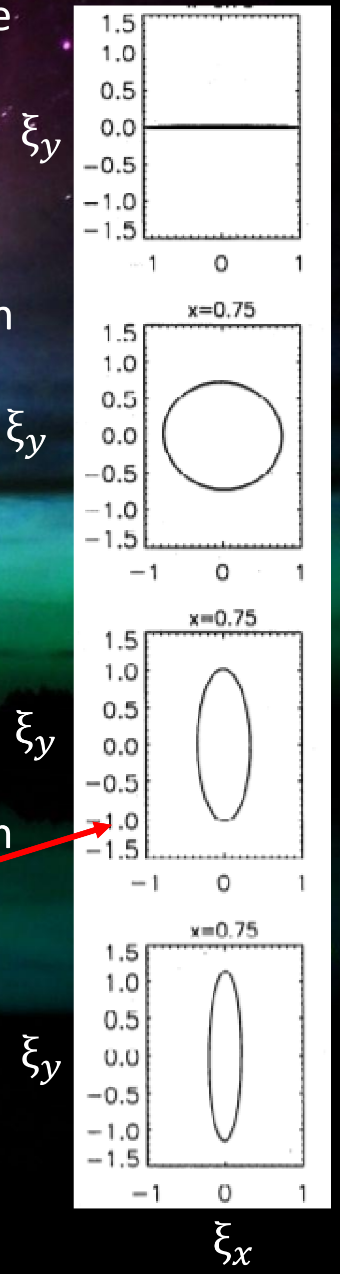
- Alfvén waves can stand along geomagnetic field lines, reflected between hemispheres by the Earth's ionosphere (Dungey, 1954).
- As transverse waves, the displacement to the magnetic field line can either be in the radial direction (poloidal) or the azimuthal direction (toroidal).
- The figure shows the first harmonic (fundamental) and second-harmonic poloidal mode structure.
- Perturbations are in the radial magnetic field component Br, corresponding to the azimuthal electric field Eϕ.
- In the Earth's curved dipole field, trapped particles are subject to a longitudinal drift (Northrop, 1963).

- The waves and particles can interact through drift resonance, when the wave and drift frequencies are harmonically related (Southwood and Kivelson, 1981,1982).



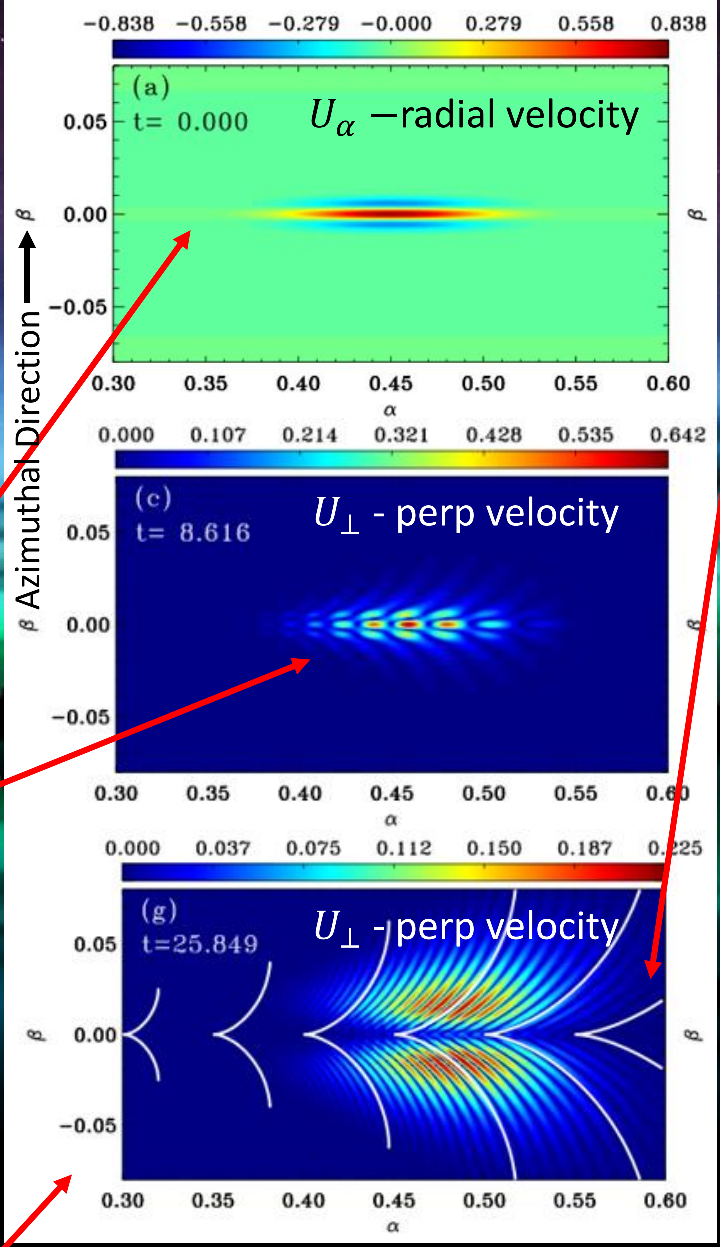
Structure of poloidal Alfvén waves (Dai+ 2013).

- Goal is to understand the spatial and temporal structure of poloidal Alfvén waves.
- It has been previously shown that in a Cartesian geometry with a straight magnetic field, that poloidal Alfvén waves with a small lengthscale azimuthally (i.e. large azimuthal wavenumber m, known as *high-m* modes) rotate their polarisation to toroidal in time (Mann+,1995).
- Displacement goes from dominantly radial (ξ_x) to azimuthal (ξ_y).
- But what happens in a curved *dipole* magnetic field?



My Work – How Poloidal Alfvén Waves Evolve in a Dipole Magnetic Field

- Perform numerical simulations using a computational MHD model with a background dipole magnetic field.
  - *__Key fact__: In a dipole magnetic field, for a given field line, the poloidal Alfvén frequency ≠ toroidal Alfvén frequency.*
  - Think of this like plucking a guitar string, if the frequency of the sound were to change depending on the direction you plucked the string – mind bending huh?
- Start with a dominantly poloidally polarised Alfvén wave, as shown in top panel – plotting contours of radial velocity, denoted U_α.
- Let this evolve in time.
- After ~6 poloidal Alfvén wave periods ($\tau_{Apol} = 1.342$), scale has reduced in radial direction due to process of phase-mixing, where neighbouring field lines drift out of phase in time (see great Youtube video of this process: https://www.youtube.com/watch?v=yVkdfJ9PkRQ&ab_channel=HarvardNaturalSciencesLectureDemonstrations)



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Radial Direction

- Final plot shows that waves have spread out along mysterious contours…
- These turn out to be contours of constant Alfvén frequency but changing polarisation.
- We can solve an equation for the Alfvén wave to determine these contours – these are the overlaid white lines.
- So the rotation to toroidal still occurs as in the Cartesian case, but now the waves have to fan out along contours of frequency.

The Tiny Summary

Curvature of dipole field causes drastic spatial effect on evolution of high-m poloidal Alfvén waves ☺.